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Relationship between the State and Subsidized Companies: Agency Problem

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ABSTRACT

This paper deals with the problem of relationship between the state and companies, which activity is subsidized by the government, given the strong influence of the state on the companies and a possibility of the state interference in the company activity. In such cases there is a conflict of interests between the state and company, i.e. depending on the subsidy level and the level of political risk for the company in the relationship due to the possibility of expropriation of funds from the cash flow controlled by company. In this case, value (utility) for one of the parties may be positive in the relationship, while for another one it may be negative. This paper deals with all possible cases of subsidy levels and expropriation parameter resulting in positive value for each party. It also deals with the issue: what conditions of subsidy level and expropriation parameter, as well as the level of efforts made by company result in value (utility) gain for each party

KEYWORDS: companies subsidized by the government, conflict of interests, political risk, agent problem, value for the state, value for the company, maximum value, cash flow.

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A large number of papers (Brealey R.A., Habib M.A. 1996; Esty B.C. 2003; Byoun S. and Xu Z. 2014) are devoted to the problem of relationship between the state and companies in project financing. In these papers, much attention is given, in particular, to the use of concession and offtake agreements grants to involve a private sponsor in project financing. Taking into account the interests of both the private sponsor and the state, the optimal level of such concessions is studied. The relationship between the state and the state subsidized company is less investigated. The state is obliged to subsidize the activity of the company, which produces socially important goods, for which it is impossible or difficult to find the substitute goods in such circumstances. Sometimes, in principle, it is possible to find a substitute, but this requires additional state investments in substitute goods, in infrastructure, without which these substitute goods will not appear, investments, financial investments in the form of concession and offtake agreements grants, etc. The costs on these additional investments may be so significant that they may be unsustainable for the state budget under the specific circumstances. The circumstances may be internal and external. The external circumstances may occur in the form of external political risks. Different kinds of external sanctions at the state and corporate level resulting in limitation of funding opportunities and difficulty of the business project implementation that require considerable efforts: financial, managerial, innovation, etc., may be the example of such risks. The adverse external circumstances may be related to the external market risks associated with the change of the market conditions for the main export goods, such as oil, gas, metals, etc.

Adverse market conditions for these goods, i.e. a sales slowdown, results in decrease in cash flow coming into the country, and the amount of tax revenue decreases accordingly. The government may then have the limitations associated with implementation of global infrastructure projects requiring significant investments. The internal circumstances include the possibility that, although this issue is important for the state, there are many other issues that may not be deferred. And this problem solution is still deferred, and this situation may last long enough. For example, the situation with activity of the energy companies supplying the beneficiary regions may serve as an example of such circumstances. These regions are often economically underdeveloped, where there is practically no large business. Sometimes these regions have a large territory and small population distributed throughout the territory.

Due to the economic underdevelopment of the region, the level of population solvency is low. At the same time, the share of population in consumption of the energy company's goods (electricity and heat) sometimes can reach up to 80%. Considering this circumstance, the government limits the company product tariffs.

For all the reasons mentioned above, the cash flow of this company is not determined and shows significant volatility. And the cash flow level is often insufficient for normal operation of the company and manufacture of products in a volume sufficient to meet the consumers' needs. Of course, the government takes into account the social needs of population and shall fulfill its social obligations. Having no economic opportunity to radically effect the outdated and inefficient energy system structure of the region, the government is obliged to somehow, albeit inefficiently subsidize the energy companies providing the region with power. Such system of relationship between the government and the subsidized company may exist long enough, until the government finds sufficient means to change significantly the way of supply of this region with corresponding goods of adequate quality and in the required quantity. The described system of relationship between the state and subsidized company creates an interesting agent problem between the government and the company (the government is a principal, the company is an agent). An emphasis on the agent problem is made in the financial literature. The agent problem was globally set and investigated in the papers of Jensen M.C., Meckling W.H. 1976, Jensen M.C. 1998, etc. The concrete mathematical models specifying the applicable contracts to mitigate this problem in case where a business owner acts as a principal and the management acts as an agent, given the risks for both sides, were investigated, for example, in the papers of Gibbons R. 2010; Gibbons R. 2005, Minasyan V. 2014. But we will be interested in the agency problem that arises between the state and the state subsidized company. If a subsidy from the government is significant, it may deprive the company of an incentive to make significant efforts to obtain good results (high cash flows), for example, by reducing costs or using any innovations. I.e. the company, represented by its top managers, will seek to an increase in utility (value) for itself and will not care about the state interests (the utility for the state) in this relationship. The government, represented by the certain officials delegated to represent the interests of the state in the relationship with the company, may temporarily decrease the amount of the subsidy already given by the government to subsidize this company by finding, from their point of view, "more important" ways to use these amounts at this stage. Allocating the appropriate amounts to subsidize the company, the appropriate officials from the government, who sometimes suffer a shortage of funds to finance other projects in this region, may from time to time ask the company to participate in financing of other projects, actually depriving the company of a part of the cash flow generated by the company. I.e. the subsidized company has a risk of expropriation of a part of the entire cash flow, which it could control in this system of relationship.

A natural task of harmonization of this complex relationship arises, taking into account the fact that, generally speaking, both members of this relationship have a specific idea of utility for themselves. In this paper, we construct a mathematical model of this relationship, taking into account the interests and risks of both the company and the state, and examine the optimal behavior of the company and the state in terms of utility for them.

Model description

Let us set the annual cash flow amount generated by the company X, X = qp, where q is the quantity of goods produced by the company per year, and p is a unit price. In view of the above mentioned state of the company, we suppose that the cash flow value is an uncertain and random value X uniformly distributed within the interval [a,b]. The company also invests its own funds in the amount of B to do its business. We suppose that the company shall have an amount not less than c per year to ensure its normal operation enabling to produce the required quantity of products (required by the government from the company). It is assumed that to increase the chances of reaching the amount of this value, the government subsidizes the company in the amount of K. If the company does not make additional efforts, the full amount available to the company is usually not enough for the normal operation of the company even in view of the subsidy Y = X + K, i.e. in this case Y < c. In such a case, there is no expropriation of funds on the part of public authorities, i.e. the company maintains control over the amount Y. If the company makes the necessary efforts, which are evaluated by value e, it is supposed that it will be able to decrease its costs by v (or increase utility for itself by v). Of course, the company will make the necessary effort, only if v > e. In such a case, these is a possibility that the entire amount, which the company has at its disposal Y = X + K, may be enough for the normal operation of the company, i.e. random value Y may also take a value of Y $\geq$ c. However, in case of such increased flows (i.e. if it turns out that Y $\geq$ c), the possibility of the state interference in the company management and expropriation of funds from the entire amount, which the company has at its disposal Y, increases. It is implemented though parameter θ in the model, where $0 \leq \theta \leq 1$. It is the parameter determining the risk of expropriation. Expropriation is expressed in a way that the amount $Y_\theta = X(1-\theta) + K\theta$ is actually left at the company's disposal. Thus, the government authorities leave the company the share θ of the subsidy amount K, and the share 1 - θ of the company's cash flow amount X. The closer θ is to one, the bigger share of the subsidy amount is left with the company. The closer θ is to zero, the bigger share of the company's cash flow amount is left. In such a case, the additional amount $Y_{1-\theta} = X\theta + K(1-\theta)$ is withdrawn by the government for other needs.

I) Value for the company without efforts

Let us consider the value (utility) for the company, if it does not make additional effort to decrease expenses, V(K). It is evident that

$V(K) = E(Y) - B = E(Y \mid Y < c) - B = E(X + K \mid X < c - K) - B$,

where an expected value symbol is expressed as E(.), and E(.|.) is a conditional expected value symbol.

I.e. in this case, receiving subsidy K, the company make no efforts, and limitation by c – K value takes place for the company's cash flow value. The more the subsidy value is, the less the cash flow generated by the company is.

The further calculations depend on the fact, whether the value c – K is more than the maximum possible value of the company's cash flow b or not.

Let us consider the first case:

1) c – K > b or K < c –b. I.e. this is the case, when the subsidy value will not cover the lack of funds for normal operation of the company, even if the company receives the maximum possible cash flow b.

In this case (see the proof in Appendix)

$$V(K) = \mu + K - B. \tag{1}$$

As is known, $\mu = E(X) = \frac{b+a}{2}$ is an expected value of the company's cash flow X. The designation $\sigma = b - a$ will be further used for the value proportional to the standard deviation of the cash flow random value X (its "volatility"), which, as is known, is $\frac{b-a}{2\sqrt{3}}$.

However, such relationship with the government is of some interest for the company, only if the value takes on the positive, V(K) >0.

This implies the need to fulfill the following inequality:

$\mu + K - B > 0$ or $K > B - \mu$. I.e. the subsidy value shall be sufficiently large. And the more the subsidy amount K is, the more the value for the company.

Let us consider the second case:

2) a$\le$c – K $\le$ b or c – a $\ge$K $\ge$ c –b. I.e. this is the case, when the subsidy value will cover the lack of funds for normal operation of the company, if the company receives the maximum cash flow level b.

In this case (see the proof in Appendix)

$$V(K) = \frac{1}{2(b-a)}(K^2 - 2(c+a-b)K + c^2 - a^2 - 2(b-a)B). \tag{2}$$

The subsidy value, with which the minimum value for the company is achieved, is defined by the equation: $2K - 2(c+a-b) = 0$. I.e. the minimum value is achieved, when

$K = K_{\min} = c - (b-a) = c - \sigma.$

The quantity of the minimum value is

$$V_{\min} = \frac{1}{2(b-a)}((b-a)^2 - a^2) + c + a - b - B =$$

$$= \frac{b-a}{2} - \frac{a^2}{2(b-a)} + c - (b-a) - B = c - B - \frac{\sigma}{2} - \frac{a^2}{2\sigma}.$$

The minimum value takes on the largest quantity depending on “volatility” σ, if volatility meets the condition:

$-\frac{1}{2}+\frac{a^2}{2\sigma^2}=0$, whence it follows that $\sigma = a$, or b =2a.

I.e. the minimum value takes on the largest value, if the maximum possible value of the company's cash flow is two times more than its minimum value.

In this case, the least value is

$V_{\min} = c - B - a.$

However, such relationship with the government is of some interest for the company, only if takes on the positive value, V(K) >0.

This implies the need to fulfill the following inequality:

$$K^2 - 2(c+a-b)K + c^2 - a^2 - 2(b-a)B > 0$$

This condition is fulfilled with all values K, if

$$(c+a-b)^2 + a^2 - c^2 + 2(b-a)B < 0,$$

or

$$2c(b-a) > (b-a)^2 + a^2 + 2(b-a)B,$$

whence it follows that

$$c > \frac{\sigma}{2} + \frac{a^2}{2\sigma} + B.$$

It means that having such necessary (required) high enough cash flow levels, any subsidies meeting the conditions c – a $\geq$K $\geq$ c –b result in positive valuableness for the company.

If the necessary (required) cash flow level meets the condition

$$c \leq \frac{\sigma}{2} + \frac{a^2}{2\sigma} + B,$$

to create the positive value for the company, the subsidy level shall meet one of the following conditions:

$K > K_{(+)}$ or $0 \leq K < K_{(-)}$,

where

$K_{(-)} = \max(K_1, 0)$, $K_{(+)} = K_2$, and

$$K_{1,2} = c-(b-a) \pm \sqrt{(c+a-b)^2 + a^2 - c^2 - 2(b-a)B} =$$

$$= c-(b-a) \pm \sqrt{(b-a)^2 + a^2 + 2(b-a)B - 2c(b-a)} =$$

$$= c - \sigma \pm \sqrt{\sigma^2 + a^2 + 2\sigma B - 2\sigma c}.$$

II) Value for the company with efforts

If the company makes any effort, then according to the described above system of relationship between the company and government, value for the company $V_e(K)$ is equal to (see Appendix)

$$V_e(K) = E(Y \mid Y \le c) + E(Y_\theta \mid \mathrm{Y} > \mathrm{c}) - \mathrm{B} + \mathrm{v} - \mathrm{e} =$$

$$= E(X) - \theta E(X \mid X > c-K) - K(1-\theta)P\{X > c-K\} + K - B + v - e. \qquad (3)$$

Further calculations depend on whether the value c - K exceeds the value of maximum possible value of company cash flow b or not.

Let us consider the first case:

1) c – K > b or K < c –b. I.e. this is the case when the subsidy value will not cover the lack of funds for normal company operation, even if the company receives the maximum possible cash flow b.

Then

$$V_e(K) = \frac{b+a}{2} + \mathrm{K} - \mathrm{B} + \mathrm{v} - \mathrm{e} = = \mu + K - B + v - e.$$

However, such relationship with the government is of some interest for the company, only if takes on the positive value, V(K) >0.

This implies the need to fulfill the following inequality:

$\mu + K - B + v - e > 0$ or $K > B + e - v - \mu.$

Thus, to obtain the positive valuableness for the company the subsidy level shall exceed the value in the right part of the last inequality.

2) a ≤ c – K ≤ b or c – a ≥ K ≥ c – b. I.e. this is the case when subsidy value will cover the lack of funds for normal company operation, if the company receives the maximum cash flow b.

Then (see Appendix)

$$V_e(K) = \frac{b+a}{2} - \frac{\theta}{2\sigma}(b^2 - c^2 + 2cK - K^2) - \frac{K(b-c+K)(1-\theta)}{\sigma} + K - B + v - e. \qquad (4)$$

In this case, the subsidy value, with which the extreme value for the company is achieved, is defined by the equation:

$$-2\theta c + 2\theta K - 2(1-\theta)(b-c) - 4(1-\theta)K + 2(b-a) = 0$$

I.e. when

$$K = K_{extr} = \frac{\theta(2c-b)+a-c}{3\theta-2}.$$

Let us define at which parameter values of risk of expropriation θ this value becomes positive. This is true, if either

A) $\begin{cases} 3\theta - 2 > 0 \\ \theta(2c-b)+a-c > 0 \end{cases}$

or

B) $\begin{cases} 3\theta - 2 < 0 \\ \theta(2c-b)+a-c < 0 \end{cases}$

Equation system A) is satisfied, if $1 > \theta > \max(\frac{2}{3}, \frac{c-a}{2c-b})$,

and equation system B) is satisfied, if $0 < \theta < \min(\frac{2}{3}, \frac{c-a}{2c-b})$.

When $1 > \theta > \max(\frac{2}{3}, \frac{c-a}{2c-b})$, the valuableness for the company $V_e(K)$ takes on a minimum value at a point $K = K_{extr} = \frac{\theta(2c-b)+a-c}{3\theta-2}$. However, when

$0 < \theta < \min(\frac{2}{3}, \frac{c-a}{2c-b})$, the valuableness $V_e(K)$ takes on a maximum value at a point

$$K = K_{extr} = \frac{\theta(2c-b)+a-c}{3\theta-2}.$$

It may also be noted that since

$$K_{extr} = \frac{2}{3} + \frac{5a-2b-c}{3\theta-2},$$

if

i) $5a-2b-c>0$, i.e. $c<5a-2b$,

when $\theta \to \frac{2}{3}+0$, $K_{extr} \to +\infty$ that means that at θ parameter values reaching to $\frac{2}{3}$ on the right, the unrestricted subsidy level is needed to achieve the extreme value for the company

(see Figure 1a).

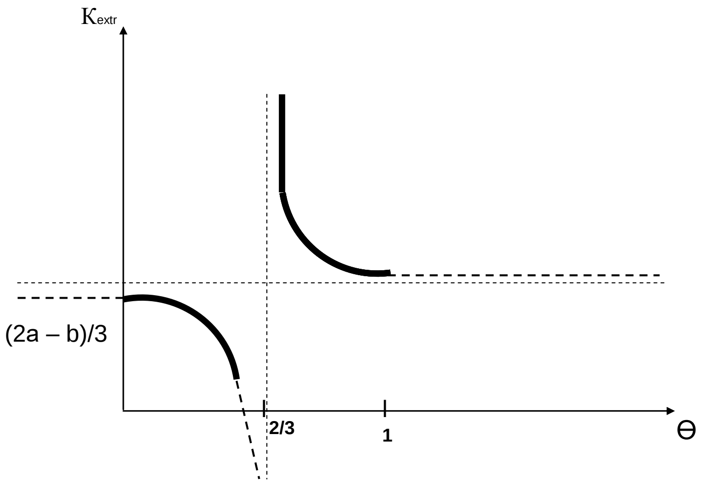

Figure 1a. Dependence of the extreme subsidy level on θ parameter. Case $c<5a-2b$.

If

ii) $5a-2b-c \le 0$, i.e. $c \ge 5a-2b$,

when $\theta \to \frac{2}{3}-0$, $K_{extr} \to +\infty$ that means that at θ parameter values reaching to $\frac{2}{3}$ on the left, the unrestricted subsidy level is needed to achieve the extreme value for the company (see Figure 1b).

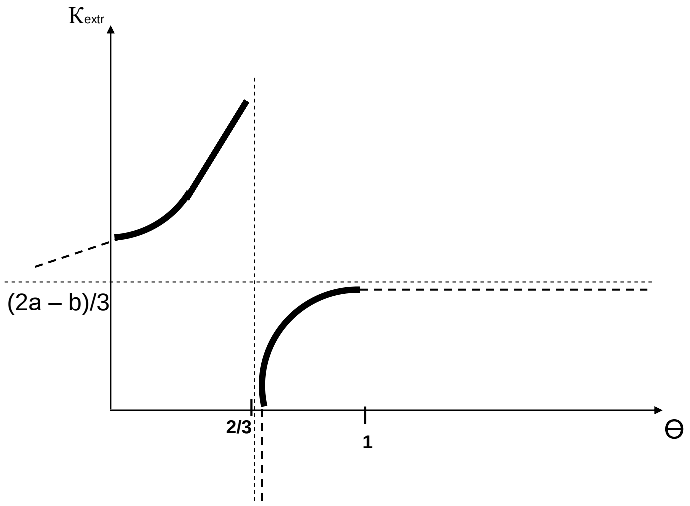

Figure 1b Dependence of the extreme subsidy level on θ parameter. Case $c \ge 5a-2b$.

However, such relationship with the government is of some interest for the company, only if takes on a positive value, $V_e(K)$ >0.

This implies the need to fulfill the following inequality:

$$(3\theta-2)K^2+2[c-a+\theta(b-2c)]K+(b-a)(b+a-2B-2e+2v)+\theta(c^2-b^2)>0. \quad (5)$$

Let us consider the following cases:

a) $3\theta-2>0$, i.e. $\theta>\frac{2}{3}$.

In this case the inequality (5) is fulfilled at any K values, if either of the following inequalities is fulfilled:

$$[c-a+\theta(b-2c)]^2-(3\theta-2)\{(b-a)(b+a-2B-2e+2v)+\theta(c^2-b^2)\}<0. \qquad (6)$$

or

$$\theta^2(2b-c)^2+\{2cb-2ab+4ac+3a^2-2c^2-5b^2+6(b-a)(B+e-v)\}\theta+$$
$$+(c-a)^2+2(b-a)(b+a-2B-2e+2v)<0$$

Which means

$$L\theta^2+M\theta+N<0, \qquad (7)$$

where

$L=(2b-c)^2\geq 0,\ M=2cb-2ab+4ac+3a^2-2c^2-5b^2+6(b-a)(B+e-v)$

and $N=(c-a)^2+2(b-a)(b+a-2B-2e+2v)$.

For the existence of θ values, at which the inequality (7) is fulfilled, it is necessary that

$M^2-4LN\geq 0.$

When fulfilling the last inequality, θ values, at which the company valuableness is positive at any K values of subsidy, are defined by inequalities

$\theta_1<\theta<\theta_2$, where

$$\theta_{1,2}=\frac{-M\pm\sqrt{M^2-4LN}}{2L}$$

However, remembering that $0<\theta<1$, we obtain the following

$\theta_{(-)}<\theta<\theta_{(+)}$,

where

$\theta_{(+)}=\min(\theta_2,1)$ and $\theta_{(-)}=\max(\theta_2,0)$.

The inequality is opposite to (6), i.e.

$$[c-a+\theta(b-2c)]^2-(3\theta-2)\{(b-a)(b+a-2B-2e)+\theta(c^2-b^2)\}\geq 0$$

it is fulfilled, if either $\theta\geq\theta_2$ or $\theta\leq\theta_1$.

Given that $0<\theta<1$, we obtain that the last inequality is fulfilled in the following cases:

A) if $\theta_2<1$, then $\theta_2\leq\theta<1$,

B) if $\theta_1 > 0,$ then $0 < \theta \le \theta_1,$

and in these cases the subsidy levels, at which the company valuableness is positive, are defined by either of the inequalities:

$K > K_2$ or $K < K_1$ where

$$K_{1,2} = \frac{-[c-a+\theta(b-2c)] \pm \sqrt{L\theta^2 + M\theta + N}}{3\theta - 2}.$$

Though, given the need to meet the natural requirement, K > 0,

it is necessary to meet the following conditions in order to fulfill $K_1 > 0$ inequality:

$$\begin{cases} c - a + \theta(b - 2c) < 0 \\ 3\theta - 2 > 0 \end{cases}$$

These conditions are met with θ parameter values satisfying the inequalities

$$\max(\frac{2}{3}, \frac{c-a}{2c-b}) < \theta < 1,$$

Otherwise, the subsidy levels, at which the valuableness for the company is positive, are defined by one inequality:

$K > K_2$

If

b) $3\theta - 2 \le 0$, i.e. $\theta \le \frac{2}{3}$,

To fulfill the inequality (5) it is necessary to fulfill the following:

$$[c-a+\theta(b-2c)]^2 - (3\theta - 2)\{(b-a)(b+a-2B-2e) + \theta(c^2 - b^2)\} \ge 0 \text{ or}$$

$$L\theta^2 + M\theta + N \ge 0 .$$

Solutions of the last inequality are defined by θ values satisfying either $\theta < \theta_1$ or $\theta > \theta_2$ inequalities,

where

$$\theta_{1,2} = \frac{-M \pm \sqrt{M^2 - 4LN}}{2L}$$

However, remembering that $0 < \theta < 1$ we obtain the following

$0 < \theta < \theta_{(-)}$ or $1 > \theta > \theta_{(+)}$,

where

$\theta_{(+)} = \min(\theta_2, 1)$ and $\theta_{(-)} = \max(\theta_2, 0)$.

In these cases, the subsidy levels, at which the valuableness for the company is positive, are defined by inequalities:

$K_1 < K < K_2$, where

$$K_{1,2} = \frac{-[c - a + \theta(b - 2c)] \pm \sqrt{L\theta^2 + M\theta + N}}{3\theta - 2}.$$

Though, given the need to meet the natural requirement, K > 0,

it is necessary to fulfill the following conditions in order to fulfill $K_1 > 0$ inequality:

$$\begin{cases} c - a + \theta(b - 2c) > 0 \\ 3\theta - 2 < 0 \end{cases}$$

These conditions are fulfilled with θ parameter values satisfying the inequalities

$$0 < \theta < \min\left(\frac{2}{3}, \frac{c - a}{2c - b}\right).$$

However, in order for the company to make some efforts, the government shall offer it such subsidy level that the following inequality to be fulfilled

$V_e(K) - V(K) > 0$.

Let us consider the first case:

1) c – K > b or K < c –b. I.e. this is the case when the subsidy value will not cover the lack of funds for normal company operation, even if company receives the largest cash flow b.

Then

$V_e(K) - V(K) = v - \mathrm{e} > 0$.

It means that the company is interested in making additional efforts at such subsidy levels.

2) a ≤ c – K ≤ b or c – a ≥ K ≥ c –b. I.e. this is the case when subsidy value will not cover the lack of funds for normal company operation, even if company receives maximum possible cash flow b. Then

$$V_e(K)-V(K)=\frac{1}{2\sigma}[b^2-a^2-\theta(b^2-c^2-2cK-K^2)-2K(b-c+K)(1-\theta)-$$
$$-(c^2-2cK+K^2-a^2)+2(v-e)(b-a)]>0$$

Or

$$3(1-\theta)K^2-2[2c-b(1-\theta)]K+(c^2-b^2)(1-\theta)-2(v-e)(b-a)<0$$

The subsidy levels satisfying the last condition exist, only if

$$[2c-b(1-\theta)]^2-3(1-\theta)^2(c^2-b^2)+6(1-\theta)(v-e)(b-a)\geq 0$$

Let us define θ parameter values at which the last inequality is fulfilled.

Having designated $\overline{\theta}=1-\theta$, this inequality will take the following form:

$$4c^2-4cb\overline{\theta}+b^2\overline{\theta}^2-3\overline{\theta}^2(c^2-b^2)+6\overline{\theta}(v-e)(b-a)\geq 0$$

$$\overline{\theta}^2[4b^2-3c^2]-2[2cb-3(v-e)(b-a)]\overline{\theta}+4c^2\geq 0 \qquad (8)$$

Let us consider the following cases:

X) $4b^2-3c^2>0$, i.e. $c<\frac{2\sqrt{3}}{3}b$. If the inequality is fulfilled:

$$[2cb-3(v-e)(b-a)]^2-4c^2(4b^2-3c^2)\geq 0,$$

the inequality (8) will be fulfilled when $\overline{\theta}>\overline{\theta}_2$ or $\overline{\theta}<\overline{\theta}_1$, where

$$\overline{\theta}_{1,2}=\frac{E\pm\sqrt{E^2-DF}}{D},$$

where $D=4b^2-3c^2$, $E=2cb-3(v-e)(b-a)$, a $F=4c^2$.

Given that $0<\overline{\theta}<1$, inequality (8) is fulfilled when

$\overline{\theta}_{\min}<\overline{\theta}<1$ and $0<\overline{\theta}<\overline{\theta}_{\max}$,

where

$\overline{\theta}_{\min}=\min(\overline{\theta}_2,1)$ and $\overline{\theta}_{\max}=\max(\overline{\theta}_1,0)$. I.e. when

$0<\theta<1-\overline{\theta}_{\min}$ and $1-\overline{\theta}_{\max}<\theta<1$.

In case of these θ parameter values, the subsidy levels, which lead to additional value for the company from the effort made, are defined by inequalities:

$K_1<K<K_2$, where

$$K_{1,2} = \frac{2c - b(1-\theta) \pm \sqrt{(2c - b(1-\theta))^2 - 3(1-\theta)^2(c^2 - b^2) + 6(1-\theta)(v-e)(b-a)}}{3(1-\theta)}.$$

If

$[2cb - 3(v-e)(b-a)]^2 - 4c^2(4b^2 - 3c^2) < 0$,

the inequality (8) will be fulfilled at any θ.

In case

Y) $4b^2 - 3c^2 \leq 0$, i.e. $c \geq \frac{2\sqrt{3}}{3}b$. It is obvious that the inequality (8) may be fulfilled only after the following inequality is fulfilled

$[2cb - 3(v-e)(b-a)]^2 - 4c^2(4b^2 - 3c^2) \geq 0$.

At that, the inequality solutions (8) are given as interval $\overline{\theta}_1 < \overline{\theta} < \overline{\theta}_2$.

Given that $0 < \overline{\theta} < 1$, the inequality (8) is fulfilled when

$\overline{\theta}_{\max} < \overline{\theta} < \overline{\theta}_{\min}$. I.e. when $1 - \overline{\theta}_{\min} < \theta < 1 - \overline{\theta}_1$.

In case of these θ parameter values, the subsidy levels, which lead to additional value for the company from the effort made, are also defined by inequalities:

$K_1 < K < K_2$, where

$$K_{1,2} = \frac{2c - b(1-\theta) \pm \sqrt{(2c - b(1-\theta))^2 - 3(1-\theta)^2(c^2 - b^2) + 6(1-\theta)(v-e)(b-a)}}{3(1-\theta)}.$$

III) Value for the state without company efforts

The state is interested in sufficient company production output q that will be presented with utility function U(q) in the model. In this case, if the company does not make efforts e, the state counts on the low quality product output q_L. Value for the state from such system of relationship with the company is presented with value G(K), where

$G(K) = E[U(q_L)] - K.$

However, the government wants the valuableness to take on a positive value, G(K) >0.

This implies the need to fulfill the following inequality:

$E[U(q_L)] > \mathrm{K}$. I.e. the expected production utility for the state shall exceed the level of the company subsidization.

IV) Value for the state with company efforts

The state is interested in sufficient company production output q that will be presented with utility function U(q) in the model. In this case, if the company makes additional efforts e, the state counts on high quality product output q_H. Given that there is expropriation of the company funds in the large cash flow, value for the state is presented with G_e(K), where

$$G_e(K) = E[U(q_H)] + E[Y_{1-\theta} \mid Y > c] - K =$$

$$= E[U(q_H)] + E[X\theta + K(1-\theta) \mid X > c - K] - K.$$

Further calculations depend on whether the value c - K exceeds the value of the maximum possible value of the company cash flow b or not.

Let us consider the first case:

1) c – K > b or K < c –b. I.e. this is the case when the subsidy value does not cover the lack of funds for normal company operation, even if the company receives the maximum possible cash flow b.

Then

$$G_e(K) = E[U(q_H)] - \mathrm{K}.$$

However, the government wants to take on a positive value, G_e(K) >0.

This implies the need to fulfill the following inequality:

$E[U(q_H)] > \mathrm{K}$. I.e. the expected production utility for the state shall exceed the level of the company subsidization.

2) $a \le c - K \le b$ or $c - a \ge K \ge c - b$. I.e. this is the case when the subsidy value covers the lack of funds for normal company operation, if the company receives the maximum cash flow b.

Then (see proof in Appendix)

$$G_e(K) = E[U(q_H)] + \frac{K}{\sigma}(b-(c-K)) + \frac{\theta}{2\sigma}(b+c-3K)(b-(c-K)) - K. \quad (9)$$

In this case, the subsidy value, with which the extreme value for the state is achieved, is defined by equation: $2(3\theta-2)K - 2[a-c+\theta(2c-b)] = 0$

I.e. when

$$K = K_{extr} = \frac{\theta(2c-b)+a-c}{3\theta-2}.$$

When $\theta > \frac{2}{3}$, this subsidy level results in the maximum value for the state, and when $\theta < \frac{2}{3}$, this subsidy level results in the minimum value for the state. I.e. the company and state interests are contending in this case.

However, the government wants to take on a positive value, $G_e(K) > 0$.

This implies the need to fulfill the following inequality:

$$2K^2 - 3\theta K^2 + 2(b-c)K + \theta(b+c)K - 3\theta(b-c)K - 2(b-a)K + \\ + \theta(b+c)(b-c) + 2(b-a)E[U(q_H)] > 0$$

or

$$(3\theta-2)K^2 - 2[a-c+\theta(2c-b)]K + \theta(c^2-b^2) - 2(b-a)E[U(q_H)] < 0.$$

Let us consider the first case:

1) $3\theta - 2 > 0$ or $\theta > \frac{2}{3}$.

Then the subsidy levels that satisfy the last condition exist, only if

$$[a-c+\theta(2c-b)]^2 - (3\theta-2)\{\theta(c^2-b^2) - 2(b-a)E[U(q_H)]\} \ge 0$$

Let us define θ parameter values, with which the last inequality is fulfilled.

This inequality is equivalent to the following inequality:

$$\theta^2(2c-b)^2-3\theta^2(c^2-b^2)+2\theta(a-c)(2c-b)+2\theta(c^2-b^2)+6\theta(b-a)E[U(q_H)]+$$
$$+(a-c)^2-4(b-a)E[U(q_H)]\geq 0$$

or

$$\theta^2(c-2b)^2+2\theta\{2ac-ab-c^2+cb-b^2+3(b-a)E[U(q_H)]\}+$$
$$+(a-c)^2+4(b-a)E[U(q_H)]\}\geq 0$$

It means

$$R\theta^2+2S\theta+T\geq 0\,, \qquad (10)$$

where

$R=(c-2b)^2$,

$S=2ac-ab-c^2+cb-b^2+3(b-a)E[U(q_H)]$,

$T=(a-c)^2+4(b-a)E[U(q_H)]$

I.e. $R=(c-2b)^2\geq 0$, if in this case inequality $S^2-RT<0$ is fulfilled, inequality (9) is fulfilled at any θ parameter values.

If opposite inequality $S^2-RT\geq 0$ is fulfilled, inequality (10) is fulfilled, when $\theta\geq\theta_2$ or $\theta\leq\theta_1$,

where

$$\theta_{1,2}=\frac{-S\pm\sqrt{S^2-RT}}{R}$$

However, remembering that $0<\theta<1$, inequality (9) is fulfilled, when $\theta_{(+)}<\theta<1$ or $0<\theta<\theta_{(-)}$

where

$\theta_{(+)}=\min(\theta_2,1)$ and $\theta_{(-)}=\max(\theta_2,0)$.

In this case, the subsidy levels necessary for positive valuableness for the state are defined by inequalities:

$K_1<K<K_2$, where

$$K_{1,2}=\frac{a-c+\theta(2c-b)\pm\sqrt{[a-c+\theta(2c-b)]^2-(3\theta-2)[\theta(c^2-b^2)-2(b-a)E[U(q_H)]]}}{3\theta-2}$$

Given that subsidy levels K are positive, such subsidy levels exist only when $K_2 > 0$ and are defined by inequalities

$K_1^+ < K < K_2^+$, where

$K_1^+ = \max(K_1, 0)$ and $K_2^+ = \max(K_2, 0)$.

In this case, if $K_2^+ = 0$, there are no subsidy levels resulting in positive value for the state.

Let us consider the second case, when

2) $3\theta - 2 \le 0$ or $\theta \le \frac{2}{3}$.

Then, if

$[a - c + \theta(2c - b)]^2 - (3\theta - 2)\{\theta(c^2 - b^2) - 2(b - a)E[U(q_H)]\} \ge 0$,

i.e. if

$\theta_{(+)} < \theta < 1$ or when $0 < \theta < \theta_{(-)}$

where

$\theta_{(+)} = \min(\theta_2, 1)$ and $\theta_{(-)} = \max(\theta_2, 0)$,

the subsidy levels necessary for positive value for the state are defined by inequalities:

$K > K_2$ or $K < K_1$, where

$$K_{1,2} = \frac{a - c + \theta(2c - b) \pm \sqrt{[a - c + \theta(2c - b)]^2 - (3\theta - 2)[\theta(c^2 - b^2) - 2(b - a)E[U(q_H)]]}}{3\theta - 2}$$

Given that subsidy levels K are positive, such subsidy levels exist only when $K_2 > 0$ and are defined by inequalities

$K > K_2^+$ or $K < K_1^+$ where

$K_1^+ = \max(K_1, 0)$ and $K_2^+ = \max(K_2, 0)$.

In this case, if $K_2^+ = 0$, there are no subsidy levels resulting in positive value for the state.

However, if the company made an effort e, the state is interested to get the valuableness gain for itself. I.e. it is desirable to fulfill the inequality:

$G_e(K) - G(K) > 0$.

Let us consider the first case:

1) c – K > b or K < c –b. I.e. this is the case when the subsidy value does not cover the lack of funds for normal company operation, even if company receives the maximum possible cash flow b.

Then

$G_e(K) - G(K) = E[U(q_H)] - E[U(q_L)] > 0$

It means that the state is interested in the company efforts, if the expected high quality product output utility is larger than that of the low quality products, i.e. $E[U(q_H)] > E[U(q_L)]$.

2) a ≤ c – K ≤ b or c – a ≥ K ≥ c –b. I.e. this is the case when the subsidy value covers the lack of funds for normal company operation, if the company receives the maximum cash flow b. Then

$G_e(K) - G(K) =$

$$= (E[U(q_H)] - E[U(q_L)]) + \frac{K}{\sigma}(K - (c-b)) + \frac{\theta}{2\sigma}(b + c - 3K)(K - (c-b)) > 0. \quad (11)$$

I.e. in this case $(K - (c-b) > 0$, if natural condition $E[U(q_H)] > E[U(q_L)]$ is fulfilled, fulfillment of inequality (11) requires the inequality: $b + c - 3K > 0$, which is equivalent to $K < \frac{b+c}{3}$. I.e. when these conditions are fulfilled, the state gets the valuableness gain from the effort made by the company.

It is also known from (10) that when conditions $E[U(q_H)] > E[U(q_L)]$ and $K < \frac{b+c}{3}$ are fulfilled, increase in θ political risk parameter results in increase in valuableness gain for the state, while growth of σ company cash flow volatility reduces the valuableness gain from the effort made by the company for the state.

Let us denote $\Delta E(U) = E[U(q_H)] - E[U(q_L)]$

Let us find all K values, with wich inequality (11) is fulfilled, i.e.

$$\Delta E(U) + \frac{K^2}{\sigma} - \frac{K}{\sigma}(c-b) - \frac{3\theta}{2\sigma}K^2 + \frac{\theta}{2\sigma}(3(c-b) + b + c)K + \frac{\theta}{2\sigma}(c^2 - b^2) > 0$$

or

$(2-3\theta)K^2 - 2(c-b-\theta(2c-b))K + \theta(c^2-b^2) + 2\sigma\Delta E(U) > 0$ (12)

The following cases are possible:

1) $2-3\theta > 0$ or $\theta < \frac{2}{3}$.

If, in this case, the following inequality is fulfilled

$(c-b-\theta(2c-b))^2 - (2-3\theta)[\theta(c^2-b^2) + 2\sigma\Delta E(U)] < 0$,

inequality (12) is fulfilled at any K subsidy level values.

The last inequality is equivalent to the following one:

$$\theta^2(7c^2-4cb-2b^2) - 2\theta(2c^2-3cb+b^2-3\sigma\Delta E(U)) +$$
$$+(c-b)^2 - 4\sigma\Delta E(U) < 0$$

Or

$A\theta^2 - 2B\theta + C < 0$ (13)

where

$A = 7c^2 - 4cb - 2b^2$,

$B = 2c^2 - 3cb + b^2 - 3\sigma\Delta E(U)$,

$C = (b-c)^2 - 4\sigma\Delta E(U)$.

Let us suggest that

A) $A = 7c^2 - 4cb - 2b^2 > 0$. It is easy to check that this condition is fulfilled, when

$c > \frac{2+3\sqrt{2}}{7}b.$

If, in this case, inequality $B^2 - AC \geq 0$ is fulfilled, inequality (13) is fulfilled, when

$\theta_1 < \theta < \theta_2$

where

$$\theta_{1,2} = \frac{-B \pm \sqrt{B^2 - AC}}{A}$$

However, remembering that $0 < \theta < 1$, inequality (13) is fulfilled, when $\theta_{(-)} < \theta < \theta_{(+)}$,

where

$\theta_{(+)} = \min(\theta_2, 1)$ and $\theta_{(-)} = \max(\theta_2, 0)$.

If the opposite inequality is fulfilled:

B) $A = 7c^2 - 4cb - 2b^2 \leq 0$, i.e. if $c \leq \frac{2 + 3\sqrt{2}}{7} b$,

inequality (13) is fulfilled, when $0 < \theta < \theta_{(-)}$ or when $\theta_{(+)} < \theta < 1$

In all these cases, the subsidy levels necessary for valuableness gain from additional efforts made by the company for the state are defined by inequalities: $K > K_2$ or $K < K_1$, where

$$K_{1,2} = \frac{c - b - \theta(2c - b) \pm \sqrt{[c - b - \theta(2c - b)]^2 - (2 - 3\theta)[\theta(c^2 - b^2) + 2\sigma\Delta E(U)]}}{2 - 3\theta}$$

Given that K subsidy levels are positive, the subsidy levels necessary for valuableness gain for the state are defined by inequalities

$0 < K < K_1^+$, where $K_1^+ = \max(K_1, 0)$ or $K > K_2$.

If

2) $2 - 3\theta \leq 0$ or $\theta \geq \frac{2}{3}$.

If, in this case, inequality

$(c - b - \theta(2c - b))^2 - (2 - 3\theta)[\theta(c^2 - b^2) + 2\sigma\Delta E(U)] \geq 0$ (14) is fulfilled,

inequality (12) is fulfilled, when K subsidy level values satisfy the conditions:

$K_1 < K < K_2$.

Given that K subsidy levels are positive, the subsidy levels necessary for valuableness gain for the state are defined by inequalities

$K_1^+ < K < K_2$ where $K_1^+ = \max(K_1, 0)$.

Let us find θ parameter values, when inequality (14) is fulfilled.

It is obvious that, when

C) $A = 7c^2 - 4cb - 2b^2 > 0$. I.e. when $c > \frac{2 + 3\sqrt{2}}{7} b$

inequality (13) will be fulfilled when $0 < \theta \leq \theta_{(-)}$ or when $\theta_{(+)} \leq \theta < 1$.

If

D) $A = 7c^2 - 4cb - 2b^2 \le 0$ or when $c \le \frac{2+3\sqrt{2}}{7}b$

inequality (13) is fulfilled, when $\theta_{(-)} < \theta < \theta_{(+)}$.

Conclusion

This paper deals with the relationship between the state and the state subsidized company. In such cases there is a conflict of interests between the state and company, i.e. depending on the subsidy level and the level of political risk for the company in the relationship due to the possibility of expropriation of funds from the cash flow controlled by company. In this case, value (utility) for one of the parties may be positive in the relationship, while for another one it may be negative.

The company can make additional efforts to increase the valuableness. But, in this case, valuableness (utility) gain from the efforts made by the company does not always result in value (utility) gain for the state. This paper deals with all possible cases of subsidy levels and expropriation parameter resulting in positive valuableness for each party. It also deals with the issue: what conditions of subsidy level and expropriation parameter, as well as the level of efforts made by company result in value (utility) gain for each party.

Appendix

Proof of formula (1)

Let us consider the first case:

If c – K > b or K < c – b,

$$V(K) = \frac{1}{\sigma}\int_a^b (x+K)dx - B = \frac{1}{\sigma}\left(\frac{b^2}{2} - \frac{a^2}{2} + K(b-a)\right) - B = \frac{b+a}{2} + K - B = \mu + K - B. \quad (1)$$

Proof of formula (2).

If a ≤ c – K ≤ b or c – a ≥ K ≥ c – b,

$$V(K) = \frac{1}{\sigma}\int_a^{c-K} xdx + K - B = \frac{1}{\sigma}(\frac{(c-K)^2}{2} - \frac{a^2}{2}) + K - B =$$

$$= \frac{1}{2(b-a)}(K^2 - 2(c+a-b)K + c^2 - a^2 - 2(b-a)B). \ (2)$$

Proof of formula (3).

If the company makes effort e, according to the described system of relationship between the company and the government, value for the company $V_e(K)$, is equal to

$$V_e(K) = E(Y \mid Y \le c) + E(Y_\theta \mid \mathrm{Y} > \mathrm{c}) - \mathrm{B} + \mathrm{v} - \mathrm{e} =$$

$$= E(X \mid X \le c-K) + KP\{X \le c-K\} + (1-\theta)E(X \mid X > c-K) + K\theta P\{X > c-K\} - B + v - e =$$

$$= E(X) - \theta E(X \mid X > c-K) + K - KP\{X > c-K\} + K\theta P\{X > c-K\} - B + v - e =$$

$$= E(X) - \theta E(X \mid X > c-K) - K(1-\theta)P\{X > c-K\} + K - B + v - e. \ (3)$$

Proof of formula (4).

$$V_e(K) = \frac{b+a}{2} - \frac{\theta}{\sigma}\int_{c-K}^{b} xdx - \frac{K(1-\theta)(b-c+K)}{\sigma} + K - B + v - e =$$

$$= \frac{b+a}{2} - \frac{\theta}{\sigma}(\frac{b^2}{2} - \frac{(c-K)^2}{2}) - \frac{K(1-\theta)(b-c+K)}{\sigma} + K - B + v - e =$$

$$= \frac{b+a}{2} - \frac{\theta}{2\sigma}(b^2 - c^2 + 2cK - K^2) - \frac{K(b-c+K)(1-\theta)}{\sigma} + K - B + v - e. \ (4)$$

Proof of formula (9).

$$G_e(K) = E[U(q_H)] + \frac{1}{\sigma}\int_{c-K}^{b}(x\theta + K(1-\theta))dx - K =$$

$$= E[U(q_H)] + \frac{1}{\sigma}(\theta(\frac{b^2}{2} - \frac{(c-K)^2}{2}) + K(1-\theta)(b-c+K)) - K =$$

$$= E[U(q_H)] + \frac{K}{\sigma}(b-(c-K)) + \frac{\theta}{2\sigma}((b-K)^2 - (c-2K)^2)) - K =$$

$$= E[U(q_H)] + \frac{K}{\sigma}(b-(c-K)) + \frac{\theta}{2\sigma}(b+c-3K)(b-(c-K)) - K. \quad (9)$$

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